Vectors

1. a) Determine $\mathbf{w} = \mathbf{u} \times \mathbf{v}$ given that $\mathbf{u} = -i + 2j + k$ and $\mathbf{v} = 3i - j + 2k$.
   
   b) What is $\mathbf{v} \times \mathbf{u}$ equal to?
   
   c) Find the angle between the two vectors $\mathbf{u} = -i + 2j + k$ and $\mathbf{v} = 3i - j + 2k$.

2. Show that the set of vectors is orthonormal
   $\left( \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, 0, \frac{1}{\sqrt{3}} \right), \left( \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, 0, \frac{1}{\sqrt{3}} \right), \left( 0, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right), \text{ and } \left( \frac{1}{\sqrt{3}}, 0, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right)$

3. Show that
   \[
   \frac{d}{dt} (\mathbf{u} \cdot \mathbf{v}) = \frac{du}{dt} \cdot \mathbf{v} + \mathbf{u} \cdot \frac{dv}{dt}
   \]
   and
   \[
   \frac{d}{dt} (\mathbf{u} \times \mathbf{v}) = \frac{du}{dt} \times \mathbf{v} + \mathbf{u} \times \frac{dv}{dt}
   \]
   (Hint, let $\mathbf{u} = u_x(t)i + u_y(t)j + u_z(t)k$, and $\mathbf{v} = v_x(t)i + v_y(t)j + v_z(t)k$)

4. a) Textbook (Levine’s 6th ed. Quantum Chemistry) Prob. 5.18 (a)
   
   b) Textbook Prob. 5.19 (a)

Angular Momentum

5. Textbook Prob. 5.29

6. Textbook Prob. 5.34

7. Prove that $\hat{L}^2$ commutes with $\hat{L}_x, \hat{L}_y$, and $\hat{L}_z$.

8. In the far infrared spectrum of H$^{79}$Br, there is a series of lines separated by 16.72 cm$^{-1}$. Calculate the values of the moment of inertia and the internuclear separation in H$^{79}$Br.

9. The following lines were observed in the microwave absorption spectrum of H$^{127}$I and D$^{127}$I between 60 cm$^{-1}$ and 90 cm$^{-1}$. Use the rigid-rotator approximation to determine the values of $\tilde{B}$, I and J for each molecule. Take the masses of H, D and $^{127}$I to be 1.008, 2.013 and 126.904 amu, respectively.

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<tr>
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<th>$\tilde{v}_{obs}$ / cm$^{-1}$</th>
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<tr>
<td>H$^{127}$I</td>
<td>64.275  77.130  89.985</td>
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<tr>
<td>D$^{127}$I</td>
<td>65.070  71.577  78.094  84.591</td>
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